Quiz ~~📷~~ 13

Show all work and circle your final answer.

1. *(8 points)* A particle is traveling along the curve $y = \sin(\pi\sqrt{x}) + \ln\pi$. When the particle is at the point $(4, \ln\pi)$, the x-coordinate is *decreasing* at a rate of 2 units per second. Find the rate at which y is changing at this time.

$$y = \sin(\pi\sqrt{x}) + \ln\pi$$

$$\frac{dy}{dt} = \cos(\pi\sqrt{x}) \cdot \frac{d}{dt}(\pi x^{1/2})$$

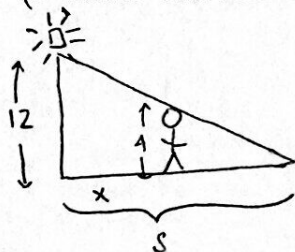$$\frac{dy}{dt} = \cos(\pi\sqrt{x}) \cdot \frac{\pi}{2\sqrt{x}} \frac{dx}{dt}$$

at $(4, \ln 4)$, $\frac{dx}{dt} = -2$:

$$\frac{dy}{dt} = \cos(\pi\sqrt{4}) \cdot \frac{\pi}{2\sqrt{4}} \cdot (-2)$$

$$= 1 \cdot \frac{\pi}{4}(-2)$$

$$= \boxed{-\frac{\pi}{2}} \text{ (units/sec)}$$

2. *(12 points)* A lamp post is 12 feet tall, and Lucy is 4 feet tall. She walks toward the lamp post at 2ft/sec. How fast is the tip of her shadow moving when she is 9 feet from the lamp post? (Please sketch and label a diagram!)

when $x = 9$ft, $\frac{dx}{dt} = -2$ ft/sec

By similar triangles,

$$\frac{s}{12} = \frac{s-x}{4}$$

$$4s = 12s - 12x \quad \text{(cross-multiply)}$$

$$4\frac{ds}{dt} = 12\frac{ds}{dt} - 12\frac{dx}{dt}$$

at $x = 9$: $\quad 4\frac{ds}{dt} = 12\frac{ds}{dt} - 12(-2)$

$$-8\frac{ds}{dt} = 24$$

$$\frac{ds}{dt} = -3 \text{ ft/sec}$$

So the tip of her shadow is moving $\boxed{3 \text{ ft/sec}}$ toward the lamp post.